It appears that the boundary layer occupies about $\frac{3}{4}$ of the shock layer, a sizable fraction. Although the foregoing calculations are very approximate, they do indicate that perhaps the Reynolds number term should be included in the forementioned boundary condition [Eq. (6) of Ref. 1]. This should present no fundamental difficulty if the quazi-linearization technique is used because although the particular boundary condition would involve a linear combination of two functions, such a combination can be treated easily in the linear theory of differential equations.

In view of the forementioned we believe that if the Reynolds number were to be retained in the boundary conditions, the validity of the analysis would probably be extended down to $Re_1 \approx 100$. Secondly, we wish to call attention to the magnetic field boundary condition of Ref. 1. The body is located at the dimensionless variable $\eta = 1$ and the dimensionless magnetic field function is given in Ref. 1 by $g(\eta = 1) = g(1) = 1$ and $g'(1) = -1$. These are the boundary conditions needed to evaluate g which determines the magnetic field H. For example, in the radial direction,

$$H_r = H_0\,[2g(\eta)/\eta^2]\cos\theta \tag{4}$$

where θ is the lateral angle. At $\theta = 0$, $\eta = 1$ we find $H_r(1) = 2H_0$ although presumably it should be $H_r(1) = H_0$. It appears as though Eq. 8 of Ref. 1 should read

$$g(1) = \tfrac{1}{2}\;(dg/d\eta)_{\eta=1} = -\tfrac{1}{2} \tag{5}$$

instead of $+1$ and -1, respectively.

The magnetic interaction parameter is

$$Q_m = (\mu_e H_0)^2 \sigma r_b/\rho_\infty v_\infty \tag{6}$$

where μ_e is the permeability, σ is the electric conductivity, and $\rho_\infty v_\infty$ are the freestream density and velocity, respectively. Thus, for any given value of Q_m, it appears that everywhere the magnetic field is twice as great as it should be. For this observation one of us (R. W. Porter) is indebted to W. Ericson and A. Maciulaitus.[5]

It appears that this apparent error can be corrected by replacing Q_m by $Q_m/4$, where Q_m is defined in Eq. (6). Thus, the graph of Ref. 1 would have an abscissa labeled $Q_m/4$ instead of Q_m. We note that this new $Q_m/4$ corresponds roughly to Bush's definition of the magnetic interaction parameter in terms of the dipole moment rather than the reference magnetic field. There is only a slight difference in that Bush used the shock radius r_s instead of the body radius r_s for the characteristic length and the point of evaluation of the reference magnetic field. Indeed, if one compares the curve attributed to Bush in Fig. 1 of Ref. 1 with Bush's tabulated values, one finds that the abscissa of this figure should be interpreted as $Q_m/4$.

Finally, we wish to comment on the large difference between the results of shock standoff at $Re = \infty$, $\epsilon = \frac{1}{10}$, and those of Bush for $Re = \infty$, $\epsilon = \frac{1}{11}$. Lighthill's result substantiates both results for zero magnetic interaction parameter[1,2] but the authors of Ref. 1 show a much larger increase in the standoff distance with nonzero magnetic parameter. This difference appears much larger than can be explained by the 10% difference in ϵ. The difference is not resolved by simply changing the abscissa of the forementioned figure, since all the curves are subject to the same change in scale. Ericson[5] has suggested that the difficulty may lie in the numerical calculations since the technique of Ref. 1 apparently fails to converge beyond a certain magnetic interaction parameter. We note that this limiting parameter does not correspond to the critical interaction parameter (evaluated at the shock) of Bush, now associated with shock layer liftoff, because the results of Bush[2] can be replotted vs the magnetic interaction parameter used in Ref. 1, and no such termination then occurs.

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Reply by Authors to R. W. Porter and A. B. Cambel

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To acknowledge the three comments by Porter and Cambel,[1] we present the following reply. First of all, in our previous short note[2] the emphasis was put upon the demonstration of the computational scheme rather than on the physics of the problem. We made the assumption that the viscous effect right behind the shock wave is not important for computational convenience only. We made the same comment in an earlier work[3] that, for small Reynolds number, the thickness of shock layer and boundary layer may be of the same order of magnitude, and the usual Hugoniot conditions may be affected. Concerning the second comment, we remark that our definition of the dimensionless parameter Q_m is different from the parameter Q defined by Bush. It is true that in Fig. 1 of Ref. 2 Bush's result is incorrectly plotted. However, the comparison of our results with those obtained by Bush is not significant in any case. Lastly, we do not understand the third comment. The results of shock standoff distances for $\epsilon = \frac{1}{11}$, $Q_m = 0$ (obtained by Bush) and for $\epsilon = \frac{1}{10}$, $Q_m = 0$ (by us) do agree with that obtained by Lighthill. Furthermore, it is reasonable to expect that near the stagnation point, $\rho/\epsilon \approx$ const for fixed freestream velocity and density, and body radius. In our discussion, we have

$$\frac{\rho(\epsilon = \frac{1}{10}, Q_m = 0)}{\rho(\epsilon = \frac{1}{11}, Q_m = 0)} = \frac{0.073}{0.067} \approx \frac{11}{10}$$

The reasonableness of the results is therefore apparent.

References

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